Problem 12-19

A train starts from rest at station A and accelerates at 0.5 m/s^2 for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at 1 m/s^2 until it is brought to rest at station B. Determine the distance between the stations.

Solution

Because the train has a constant acceleration in the three parts of its motion, the kinematic formula,

$$x = x_0 + v_0 t + \frac{1}{2}at^2,$$

can be used each time to find the distance it travels.

Part 1

For the first 60 seconds, it travels

$$x = 0 + 0 + \frac{1}{2}(0.5)(60)^2 = 900 \text{ m}.$$

The train's velocity after accelerating like this for 60 seconds is $(0.5)(60) = 30$ m/s.

Part 2

After the next 15 minutes, or 900 seconds, the train's distance from station A is

$$x = 900 + 30(900) + \frac{1}{2}(0)(900)^2 = 27\,900 \text{ m}.$$

The acceleration is zero, so the train's velocity is still 30 m/s at the end of part 2.

Part 3

Since we don't know how long the train decelerates for, it's more convenient to use the kinematic formula without time, namely

$$v^2 - v_0^2 = 2a(x - x_0).$$

Solve for x, the final distance.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 27\,900 + \frac{0^2 - 30^2}{2(-1)} = 28\,350 \text{ m}$$

Therefore, to three significant figures, the distance from station A to station B is 28.4 km.